Mechanised Semantics for Gated Static Single Assignment

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Overview

1. Refresher on SSA
2. Translation from SSA to GSA
3. Proof of SSA to GSA Translation
4. Summary

Refresher on SSA

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Introduced in late 80’s [Alpern et al., 1988]

Now widely adopted in compiler community
GCC, LLVM, Java HotSpot JIT…

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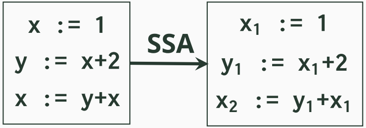
Straight-line code

Definitions: fresh variable, version number

Uses: rename variable, pick right version



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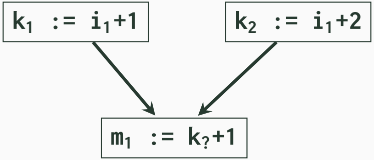

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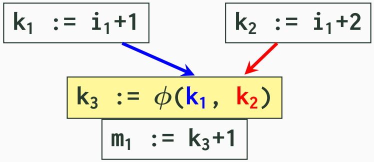
Dedicated instruction: $k_3 := \phi(k_1, k_2)$
Based on control-flow, select right argument



Benefits and Shortcomings of SSA

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Non-local semantics of ϕ-instructions

Gated SSA: Use gates to turn control into data-dependencies

Local execution of gates replacing ϕ-instructions

Gated SSA: New Instructions

Gated SSA: extends ϕ-instructions with gates

Simple join points:

$$r_d \leftarrow \gamma(\overrightarrow{(p_i, r_i)})$$

Predicate p_i discriminate arguments, local choice

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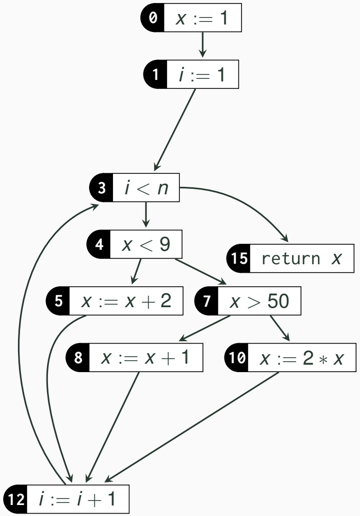
Our Contributions

- Describe a specification and control-flow semantics for Gated SSA.
- Focus on the control-flow independent semantics of gates.
- Describe implementation and proof in CompCertSSA.

Translation from SSA to GSA

Gated SSA (GSA): Example Generation



RTL

Control-flow graph for the following program:

```
int f(int n) {
  int x = 1;
  for (int i = 1; i < n; i++)
    if (x < 9) x = x + 2;
    else if (x > 50) x = x + 1;
    else x = 2 * x;
  return x;
}
```

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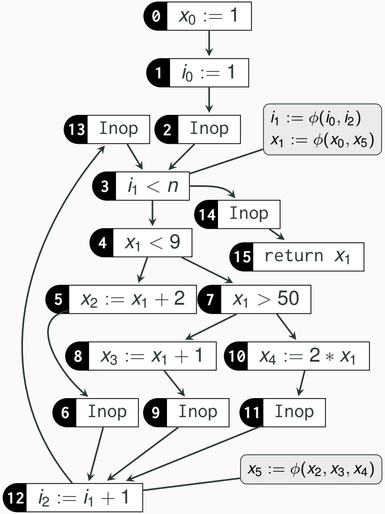

SSA

- Additional nop instructions are inserted to normalise control-flow graph.
- Variable assignments are made unique.
- Existing SSA Generation inserts ϕ-instructions.

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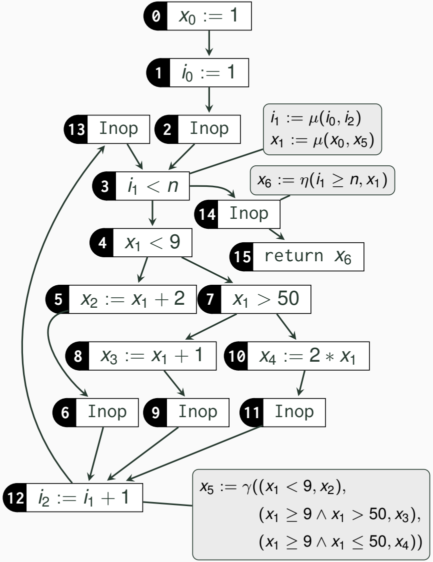

GSA

- Replace ϕ-instructions by μ- and γ-instructions, then insert η-instructions.
- Predicates use normal syntactic elements.

Translating from SSA to GSA

Single-source path expression problem

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- We translate path expressions to predicates.
- Path expression $P(s, v)$ become predicate $\mathbf{P}_{s,v}$.

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Single-source path expression problem

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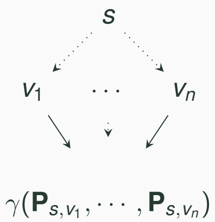
For every future γ node, get a path-expression from the dominator s to each of its predecessors v_1, v_2, ..., v_n.



Proof of SSA to GSA Translation

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- Path expression algorithm is not formalised.
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Main issues

- Reasoning about predicates is global and dynamic.
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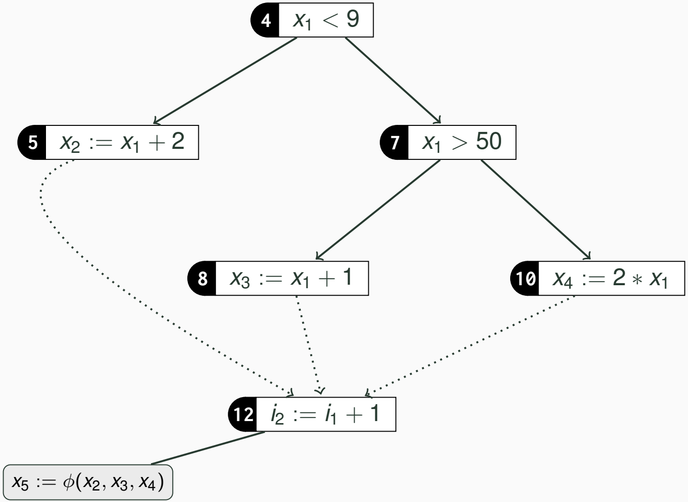
Key intuition

- Build local correctness rules about predicates for every node.
- Use them to build a proof about the evaluation of predicates.
- *Key properties*: coherence $\wedge$ mutual independence $\implies$ validity.

Predicate Generation: Example



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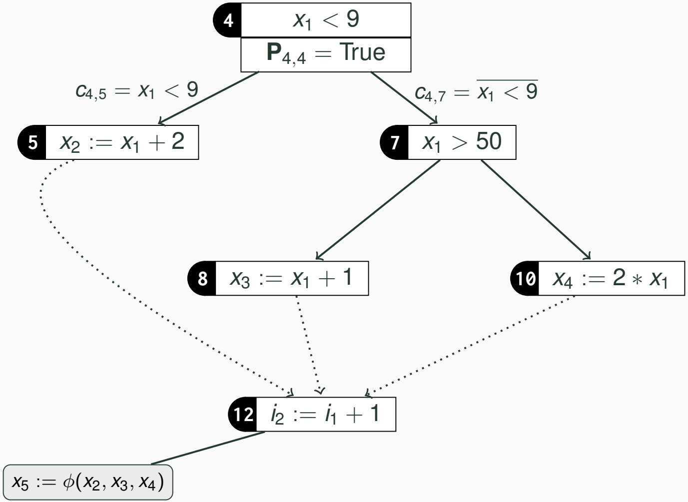

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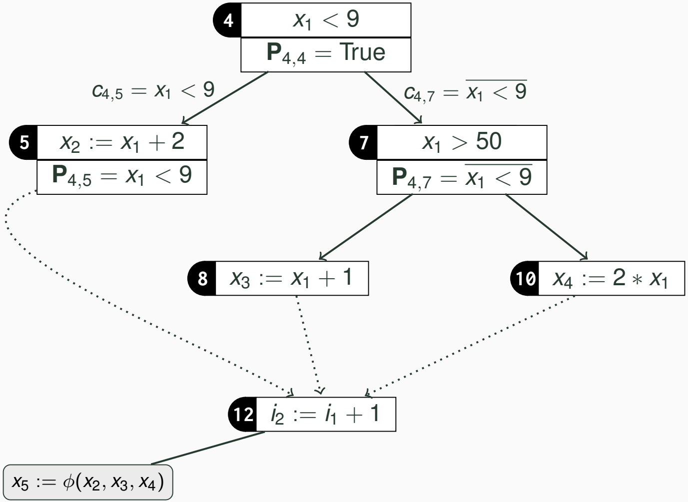

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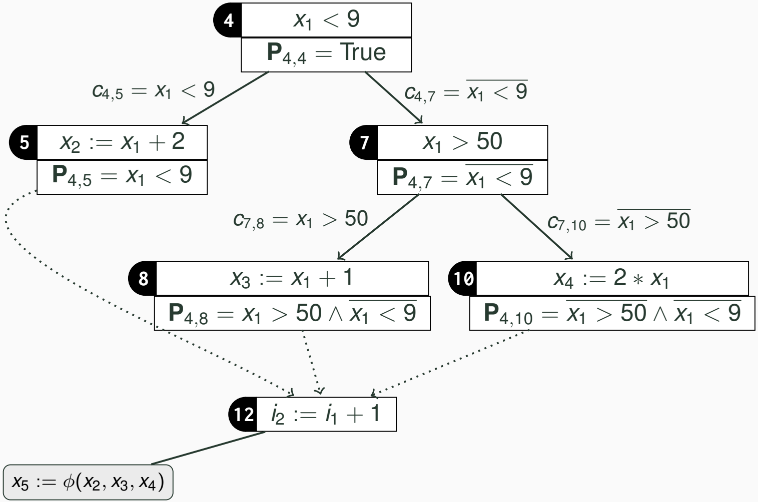

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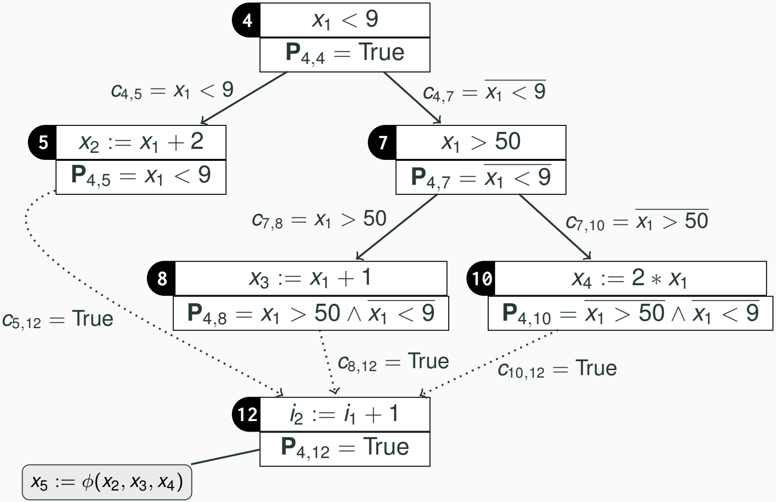
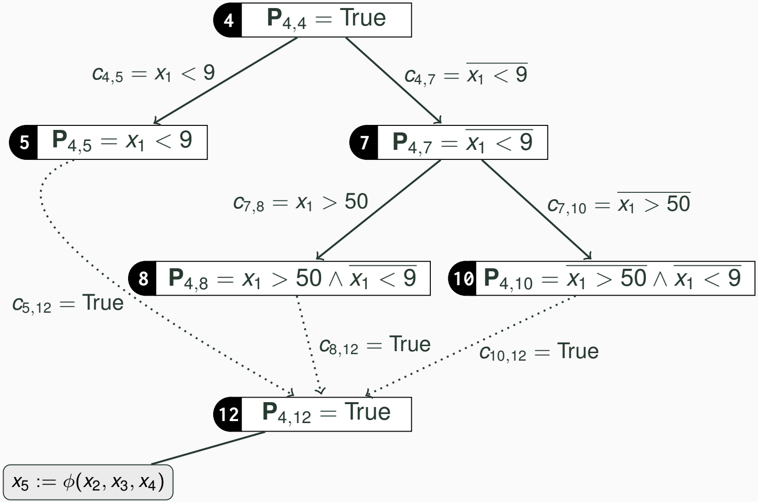


Coherence Property: Example



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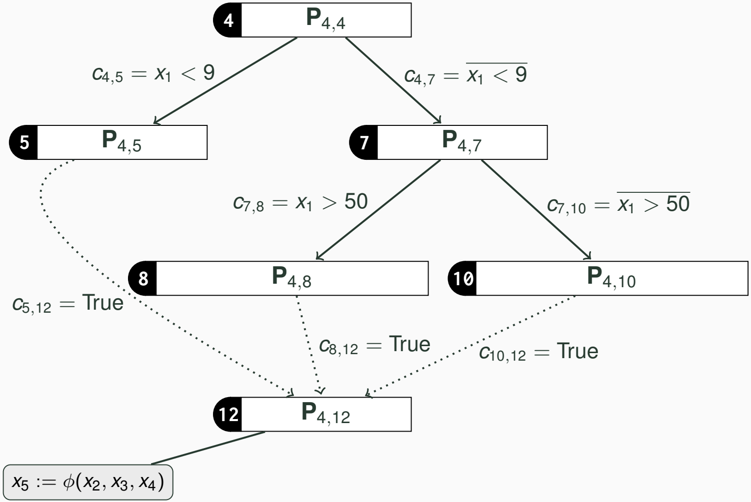

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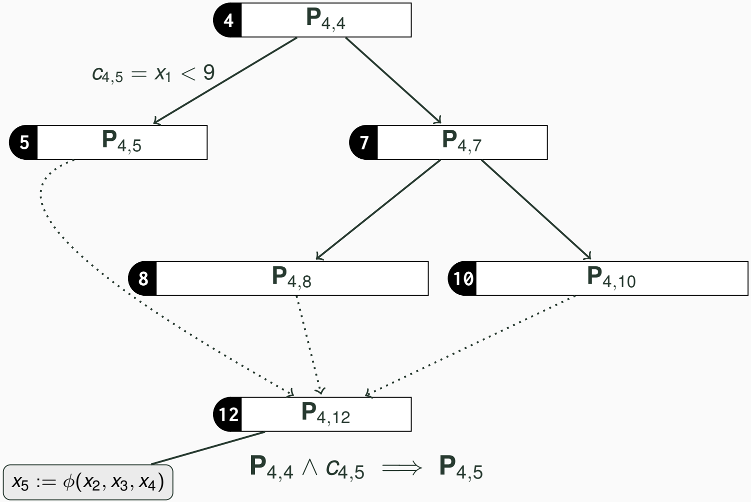
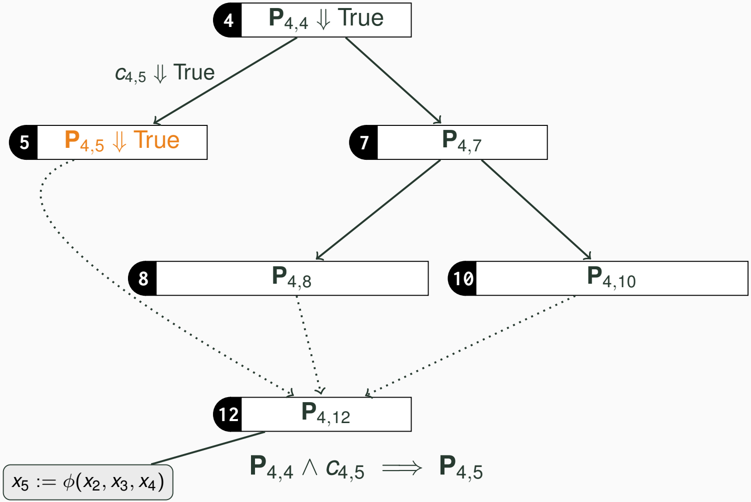


$$P_{4,4} \wedge c_{4,5} \implies P_{4,5}$$

Validity Property: Example



$$\mathbf{P}_{4,4} \wedge c_{4,5} \implies \mathbf{P}_{4,5}$$

Invariant: $\mathbf{P}_{4,i} \Downarrow \mathit{True}$

Using an SMT Solver to Check Properties

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Syntactic elements in predicates might not be evaluable.

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Summary

Summary and Future Work

Implementation within CompCertSSA

- Gated SSA: syntax and semantics
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Future work: Pure data-flow semantics, proof of Tarjan's SSPE, well-formed GSA.

Thank You, Any Questions?

Paper

Artefact

Gated SSA: New Instructions

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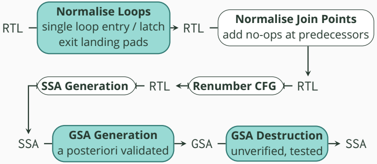

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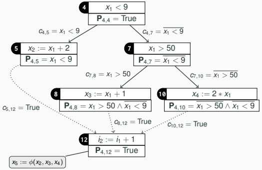

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13

Semantics of Gated SSA

Eta

$$\frac{i = r_d \leftarrow \eta(q, r) \qquad rs \models_p q \Downarrow 1 \qquad b_\eta \vdash rs \overset{\mathcal{E}}{\rightsquigarrow} rs'}{\lfloor i :: b_\eta \rfloor \vdash rs \overset{\mathcal{E}}{\rightsquigarrow} rs'[r_d \mapsto rs(r)]}$$

Merge$_\gamma$

$$\frac{i = r_d \leftarrow \gamma(\overrightarrow{(q, r)}) \qquad rs \models_p q_n \Downarrow 1 \qquad b_\mathcal{M}, k \vdash rs \overset{\mathcal{M}}{\rightsquigarrow} rs'}{i :: b_\mathcal{M}, k \vdash rs \overset{\mathcal{M}}{\rightsquigarrow} rs'[r_d \mapsto rs(r_n)]}$$

Merge$_\mu$

$$\frac{i = r_d \leftarrow \mu(r_0, r_1) \qquad k \in \{0, 1\} \qquad b_\mathcal{M}, k \vdash rs \overset{\mathcal{M}}{\rightsquigarrow} rs'}{i :: b_\mathcal{M}, k \vdash rs \overset{\mathcal{M}}{\rightsquigarrow} rs'[r_d \mapsto rs(r_k)]}$$

NJoin

$$\frac{f.\mathcal{I}(l) = \lfloor \texttt{Inop}(l') \rfloor \qquad f \not\curlyvee l' \qquad f.\mathcal{E}(l) \vdash rs \overset{\mathcal{E}}{\rightsquigarrow} rs'}{\vdash \mathcal{S}(f, l, rs) \rightarrow \mathcal{S}(f, l', rs')}$$

Join

$$\frac{f.\mathcal{I}(l) = \lfloor \texttt{Inop}(l') \rfloor \qquad f \curlyvee l' \qquad f.\mathcal{M}(l') = \lfloor b_\mathcal{M} \rfloor \qquad f.\mathcal{E}(l) \vdash rs \overset{\mathcal{E}}{\rightsquigarrow} rs' \qquad \text{preds}(l')_k = l \qquad b_\mathcal{M}, k \vdash rs' \overset{\mathcal{M}}{\rightsquigarrow} rs''}{\vdash \mathcal{S}(f, l, rs) \rightarrow \mathcal{S}(f, l', rs'')}$$

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