EE2-08C - Numerical Analysis of ODEs/PDEs using Matlab - Coursework 2017

The coursework is due for submission on Monday 13 March 2017 either to me or the UG office room 608, before 4pm. You should hand in a *printed* copy of all your matlab files with thorough comments as well as the plots you generate, clearly labelled. Before 22:00pm on Monday 13 March, you should also submit the pdf of the report as well as the matlab files *separately* on Blackboard. Each group should submit only once: choose a member of your group to submit all items on behalf of the group.

1 RL circuit

See email, 31 Jan.

2 RLC circuit

See email, 31 Jan.

3 Finite Differences for PDE

The 1-D heat equation

$$\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

with zero boundary conditions $y(0,t) = y(1,t) = 0$, and initial condition $y(x,0) = y_0(x)$ can be solved numerically using the finite difference method outlined in lectures.

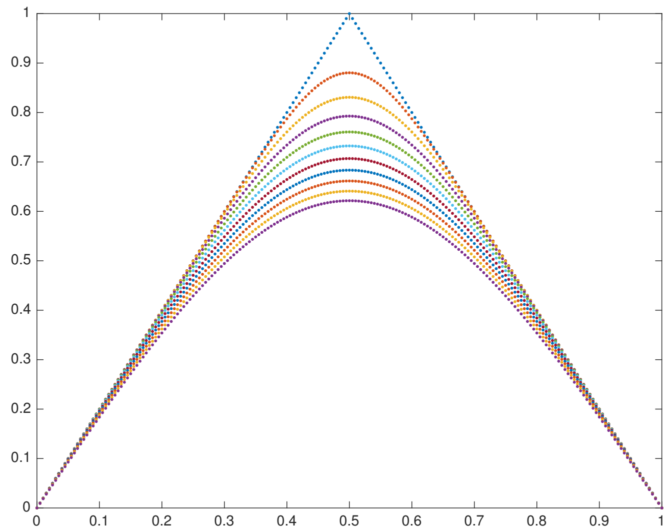
Exercise 4. Write a matlab script called **finite_script.m** to implement the finite difference method, and solve the heat equation. The initial condition given by $y(x,0) = y_0(x)$ should be tested for the two basic cases seen in lectures, a tent function and a sinusoidal function:

$$(i)\, y_0(x) = \begin{cases} 2x, & \text{for } x \in [0, 0.5] \\ 2 - 2x & \text{for } x \in [0.5, 1] \end{cases} \qquad \text{and} \qquad (ii)\, y_0(x) = \sin(2\pi x)\,.$$

Test it also for $y_0(x) = |\sin(2\pi x)|$. Can you explain the behaviour you observe? Finally, test it for two further initial conditions of your choice. Be sure to supply plots.

Some points you need to consider:

- How to allocate the boundary conditions U_0^m and U_N^m;
- How to implement the central algorithm $U_j^{m+1} = vU_{j-1}^m + (1-2v)U_j^m + vU_{j+1}^m$;
- How to choose h, k;
- How to plot U_j^m for a sensible choice of values of m, to be able to visualize progress as time increases. Here is what you expect for the tent function: (PTO)



Each set of coloured dots represents U_j^m for one value of m and $j = 0 \ldots N$. The top set is the initial condition; as time increases, the values all decrease.

Bonus Exercise You can get full marks by answering all of the above. If you want to improve your mark, you may try any of the following, but the maximal mark will still be 100%.

(i) Extend your work on the PDE solution to include an initial condition that does not match one or both boundary conditions.

(ii) Further extend by including constant, non-zero boundary conditions. (iii) Further extend to include time-varying boundary conditions.